Basic model for "The Role of Social Norms in Old-age Support: Evidence from China"

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0.1 Baseline model

The model describing the same-gender demonstration effect in the following section is based on the demonstration effect model by Cox and Stark (1996, 2005), combined with a definition of intergenerational transfers taken from a model by Banerjee et al. (2014). It is a simple inter-temporal two-period consumption model. Cox and Stark (1996, 2005) maintain that "... childhood experience affects behaviour in adulthood". Parents who value support for the elderly will demonstrate the norm of providing support for the elderly to their children by providing support to their own elderly parents. Based on the demonstration effect, the model assumes that parents know that their support to their own elderly parents will affect the future support behaviour of their same-gender children. Another assumption noted above is that children will be affected by the behaviour of their same-gender parents. Given differences in anticipation of the future and same-gender intergenerational transmission, the model predicts that parents will provide support to their own parents, according to the gender of their children. This explains the relationship between parents' support for the elderly and the gender ratio of their children.

There are three generations in the model: the mid-age generation (P), the parents; the older generation (O), parents of P, and the younger generation (K), children of P. They correspond to the second generation, the first generation and the third generation respectively, but only in this paper. There are two periods in the model: the first period, $t = 1$, and the second period, $t = 2$. The baseline model uses the notation in Banerjee et al. (2014) and requires a few additional assumptions:

- (i) each household in P has a father and a mother;
- (ii) the father transfers a fraction τ_1^F of his income and the mother transfers a fraction τ_1^M of hers to their own parents. Both of them have income Y_1. Y_1 is exogenous;
- (iii) the number of K in each household, n, is exogenous. The male-to-female gender ratio of children in a household is ϕ;

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- (iv) people value their parents' welfare as well as their own consumption, so they derive utilities from providing transfers to their parents. However, there is also a discount factor, $0 < \delta < 1$, for the utility derived from the provision of old-age support, since the transfer to O is not direct consumption for the individuals;

- (v) τ_t^F and τ_t^M are endogenous and different when $t = 1$ and when $t = 2$. The transfer from the children of the father and mother in the second period will be affected by their same-gender parents' transfer in the first period.[1] In the equations, this assumption is expressed as

$$\tau_2^F = \mathcal{T}^F(\tau_1^F) \quad \text{and} \quad \tau_2^M = \mathcal{T}^M(\tau_1^M). \tag{1}$$

  Both functions are strictly concave and increasing in τ_1^F and τ_1^M, and

$$\tau_2^F = 0 \quad \text{if} \quad \tau_1^F = 0 \quad \text{and} \quad \tau_2^M = 0 \quad \text{if} \quad \tau_1^M = 0;$$

- (vi) the father and the mother in a household make unitary household-level decisions. The household consumption is c_t in each time period;

- (vii) for simplicity, I assume the transfer from P to their parents-in-law would only make their children provide transfers to their parents-in-law in the second period. So providing transfers to P's parents-in-law is not in line with the interest of the P's household. So I do not consider the transfer to P's parents-in-law here;[2]

- (viii) for simplicity, I assume that there is no saving in the baseline model;[3]

- (ix) $u(\cdot)$ is a strictly concave function.

In this model, P is the generation solving the optimisation problem in the first period. O passively receives support from P in the first period and dies in the second period. Members of K observe their parents' τ_1 in the first period and provide their parents with τ_2 in the second period. With the assumptions above, a typical household in generation P solves the following problem:

$$\max_{\tau_1^F, \tau_1^M} \quad U = u(c_1) + \delta u(e_1) + \beta u(c_2)$$

$$s.t.$$

$$c_1 + c_2 \leq Y_1(2 - \tau_1^F - \tau_1^M) + Y_2(\mathcal{T}^F(\tau_1^F)\phi n + \mathcal{T}^M(\tau_1^M)(1-\phi)n);$$

$$e_1 = Y_1(\tau_1^F + \tau_1^M).$$

[1] Household bargaining parameter is not included in the basic model for simplicity. It is included in the model not presented here. The results show that if the males' income is much larger than the females', the corresponding father conclusion holds. If the females' income is much larger than the males', the corresponding mother conclusion holds.

[2] This assumption is a bit restrictive. I should consider incorporating the relaxed version of this assumption in future.

[3] Saving is not included in the basic model for simplicity. The saving parameter is included in the model not presented here. The results show that under certain reasonable conditions, the corresponding conclusions holds.

The father and the mother in generation P make unitary household-level decisions, and there is no saving, thus that the expressions for the household consumption for the two periods are as follows:

$$c_1 = Y_1(2 - \tau_1^F - \tau_1^M); \quad c_2 = Y_2[\mathcal{T}^F(\tau_1^F)\phi n + \mathcal{T}^M(\tau_1^M)(1-\phi)n].$$

e_1 is the old-age support provided by the whole household. δ is the discount factor for the utility generated from altruism, and β is the time discount factor. If $u(c)$ is specified as a log or a CRRA function, and τ_2 is a concave function of τ_1, the FOCs with respect to τ_1^F and τ_1^M are:

$$U^1 = \frac{\mathrm{d}U}{\mathrm{d}\tau_1^F} = u'(c_1)(-Y_1) + \delta u'(Y_1(\tau_1^F + \tau_1^M))Y_1 + \beta u'(c_2)Y_2\tau_2^{F'}\phi n = 0; \tag{2}$$

$$U^2 = \frac{\mathrm{d}U}{\mathrm{d}\tau_1^M} = u'(c_1)(-Y_1) + \delta u'(Y_1(\tau_1^F + \tau_1^M))Y_1 + \beta u'(c_2)Y_2\tau_2^{M'}(1-\phi)n = 0. \tag{3}$$

Given Equations (2) and (3), I obtain the following condition to derive the optimal τ_1^F and τ_1^M, which are τ_1^{F*} and τ_1^{M*} respectively:

$$\frac{\tau_2^{F'}}{\tau_2^{M'}} = \frac{1-\phi}{\phi}. \tag{4}$$

From the FOCs, I can derive the SOCs corresponding to τ_1^F, τ_1^M, and ϕ. Recall that $c_1 = Y_1(2 - \tau_1^F - \tau_1^M)$ and $c_2 = Y_2(\tau_2^F\phi n + \tau_2^M(1-\phi)n)$. From Equation (2), the SOCs with respect to τ_1^F and ϕ are:

$$\begin{aligned}\frac{\mathrm{d}^2U}{\mathrm{d}\tau_1^{F2}} &= u''(c_1)(Y_1^2) + \delta u''(Y_1(\tau_1^F + \tau_1^M))Y_1^2 \\ &\quad + \beta u'(c_2)Y_2\tau_2^{F''}\phi n + \beta u''(c_2)(Y_2\tau_2^{F'}\phi n)^2; \\ \frac{\mathrm{d}^2U}{\mathrm{d}\tau_1^F\mathrm{d}\phi} &= \beta u''(c_2)(Y_2^2\phi n^2)\tau_2^{F'}(\tau_2^F - \tau_2^M) + \beta u'(c_2)Y_2\tau_2^{F'}n.\end{aligned} \tag{5}$$

I assign:

$$U^{11} = \frac{\mathrm{d}^2U}{\mathrm{d}\tau_1^{F*2}}; \qquad U^{13} = \frac{\mathrm{d}^2U}{\mathrm{d}\tau_1^{F*}\mathrm{d}\phi},$$

which are the SOCs at the optimal value of τ_1^F and τ_1^M. Recall that function u is strictly concave in c_1 and c_2. $\mathcal{T}^F$ and $\mathcal{T}^M$ are both strictly concave functions. U^{11} is always smaller than 0 under these assumptions. For the sign of U^{13}, when the function $u(\cdot)$ is specified as a log or a CRRA function, I obtain

$$|u''(c_2)(Y_2^2\tau_2^{F'}\phi n)(n\tau_2^F - n\tau_2^M)| < |u'(c_2)Y_2\tau_2^{F'}n| \quad \Rightarrow \quad U^{13} > 0.$$

From Equation (3), the corresponding SOCs are:

$$\frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{M2}} = u''(c_1)(Y_1^2) + \delta u''(Y_1(\tau_1^F + \tau_1^M))Y_1^2$$
$$+ \beta u'(c_2)Y_2\tau_2^{M''}(1-\phi)n + \beta u''(c_2)(Y_2\tau_2^{M'}(1-\phi)n)^2; \tag{6}$$
$$\frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^M \mathrm{d}\phi} = \beta u''(c_2)(Y_2^2(1-\phi)n^2)\tau_2^{M'}(\tau_2^F - \tau_2^M) - \beta u'(c_2)Y_2\tau_2^{M'}n.$$

The SOC for τ_1^F and τ_1^M is:

$$\frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^F \mathrm{d}\tau_1^M} = u''(c_1)(Y_1^2) + \delta u''(Y_1(\tau_1^F + \tau_1^M))Y_1^2 + \beta u''(c_2)Y_2^2\tau_2^{F'}\tau_2^{M'}\phi(1-\phi)n^2. \tag{7}$$

Here again I specify

$$U^{22} = \frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{M*2}}; \qquad U^{23} = \frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{M*}\mathrm{d}\phi}; \qquad U^{12/21} = \frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{F*}\mathrm{d}\tau_1^{M*}};$$

which are the SOCs at the optimal value of τ_1^F and τ_1^M. Because of the concave assumptions for $u(\cdot)$, $\mathcal{T}^F$, and $\mathcal{T}^M$, I infer the signs of U^{22}, U^{23}, and $U^{12/21}$ are negative, and do not depend on the specification of the utility function $u(c)$, as long as $u(c)$ is concave. If Equation (4) is substituted for Equations (5), (6) and (7), then the comparison between the absolute values of U^{11}, U^{22}, and U^{12} is

$$|U^{11}| > |U^{12}|; \qquad |U^{22}| > |U^{12}|.$$

According to the assumption of the demonstration effect, I would expect the optimal value of the transfer from the father, τ_1^{F*}, to be positively affected by his children's gender ratio, ϕ, and the optimal value of the transfer from the mother, τ_1^{M*}, would be negatively affected by ϕ. In other words, the expected comparative statics from the optimisation problem are:

$$\frac{\mathrm{d}\tau_1^{F*}}{\mathrm{d}\phi} > 0; \qquad \frac{\mathrm{d}\tau_1^{M*}}{\mathrm{d}\phi} < 0.$$

To obtain these two comparative statics, I need to totally differentiate Equations (2) and (3), which are:

$$\begin{aligned} U^{11}\mathrm{d}\tau_1^{F*} + U^{12}\mathrm{d}\tau_1^{M*} + U^{13}\mathrm{d}\phi = 0; \\ U^{21}\mathrm{d}\tau_1^{F*} + U^{22}\mathrm{d}\tau_1^{M*} + U^{23}\mathrm{d}\phi = 0, \end{aligned} \tag{8}$$

where again

$$U^{11} = \frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{F*2}}; \quad U^{13} = \frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{F*}\mathrm{d}\phi}; \quad U^{22} = \frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{M*2}}; \quad U^{23} = \frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{M*}\mathrm{d}\phi}; \quad U^{12/21} = \frac{\mathrm{d}^2 U}{\mathrm{d}\tau_1^{F*}\mathrm{d}\tau_1^{M*}}.$$

The asterisks denote optimal values. The U^{ij}s are the SOCs when $\tau_1^F = \tau_1^{F*}$ and $\tau_1^M = \tau_1^{M*}$, $i \in \{1,2\}$ and $j \in \{1,2,3\}$. Hence, the comparative statics from the conditions in Equation (8) are:

$$\frac{d\tau_1^{F*}}{d\phi} = \frac{U^{12}U^{23} - U^{13}U^{22}}{U^{11}U^{22} - U^{12}U^{21}}; \quad \frac{d\tau_1^{M*}}{d\phi} = \frac{U^{11}U^{23} - U^{13}U^{21}}{U^{12}U^{21} - U^{11}U^{22}}.$$

The signs for SOCs when $\tau_1^F = \tau_1^{F*}$ and $\tau_1^M = \tau_1^{M*}$ are:

$$U^{11} < 0; \quad U^{13} > 0; \quad U^{22} < 0;^{4}$$

$$U^{23} < 0; \quad U^{12} = U^{21} < 0.$$

From the equations for SOCs, I can obtain the sign of the numerators and denominators in the comparative statics:

$$U^{12}U^{23} - U^{13}U^{22} > 0;$$

$$U^{11}U^{23} - U^{13}U^{21} > 0;$$

$$U^{11}U^{22} - U^{12}U^{21} > 0,$$

and thus the signs of the comparative statics are:

$$\frac{d\tau_1^{F*}}{d\phi} = \frac{U^{12}U^{23} - U^{13}U^{22}}{U^{11}U^{22} - U^{12}U^{21}} > 0; \quad \frac{d\tau_1^{M*}}{d\phi} = \frac{U^{11}U^{23} - U^{13}U^{21}}{U^{12}U^{21} - U^{11}U^{22}} < 0. \tag{9}$$

The comparative statics can be summarised in the following proposition:

Proposition 1: *In the model in this section, when the utility function is specified as a log or a CRRA function, then τ_1^{F*} is increasing in ϕ and τ_1^{M*} is decreasing in the gender ratio of K, ϕ. The model shows:*

$$\frac{d\tau_1^{F*}}{d\phi} > 0; \quad \frac{d\tau_1^{M*}}{d\phi} < 0.$$

The first interpretation of the comparative statics in *Proposition 1* is that the fraction of the father's income transferred to his parents increases with the male-to-female gender ratio of his children. It also means that he will provide more old-age support to his parents the more sons he has in his household, fixing the number of K. The mother will transfer more to her own parents if she has more daughters, regardless of whether τ_1^F is greater or smaller than τ_1^M. As noted above, it is more usual in China for males to support their parents than for females. $\tau_1^F > \tau_1^M$ indicates that the father transfers more than the mother does, as a general social norm. However, the condition $\tau_1^F > \tau_1^M$ does not affect the conclusion of the baseline model.

One key assumption for the interpretations is that ϕ should be exogenous. To make sure that ϕ, the gender ratio of the generation K, is exogenous at the household-level in the empirical part of the empirical part, I use the policy change which started in 2003. From this date, the selection of unborn children by sex was banned in China. The regulation brought the gender ratio of newborns after 2003 closer to the natural rate than the gender ratio was before the policy changed.

[4] Note that $U^{13} > 0$ when the utility function is specified as a log or a CRRA function. For example, if $u(c) = log(c)$, then $U^{13} = \frac{\beta Y_2^2 n^2 \tau_2^{F'} \tau_2^M}{C_2^2} > 0$.

References

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